Z-scan experiment with anisotropic Gaussian Schell-model beams

Yongxin Liu, Jixiong Pu,* and Hongqun Qi

Department of Electric Science & Technology, Huaqiao University, Quanzhou, Fujian 362021, China
*Corresponding author: jixiong@hqu.edu.cn

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We analyze the z-scan experiment with anisotropic Gaussian Schell-model (AGSM) beams. The expression for the cross-spectral density of the AGSM beam passing through the lens and onto the nonlinear thin sample is derived. Based on the expression, we simulate the results of the z-scan experiment theoretically and analyze the effects of the \( e \) factor \((e = w_0/y/w_0)\) and the spatial degree of coherence in the \( x \) and \( y \) orientations on the on-axis z-scan transmittance. It is found that \( \Delta T_{p,z} \) becomes larger with an increment of the \( e \) factor and the spatial degree of coherence. So we can improve the sensitivity of the z-scan experiment by increasing the \( e \) factor and the spatial degree of coherence. The results are helpful for improving the sensitivity of the z-scan experiment. © 2009 Optical Society of America

1. INTRODUCTION

It is well known that z-scan has been an effective method in measuring the nonlinear coefficient of various media. This technique is based on the principles of spatial beam distortion but offers simplicity as well as very high sensitivity. We can derive the absorptive and refractive nonlinear optical properties of matter through the normalized transmittance curve in the case of open aperture and closed aperture [1–4]. Z-scan is better than previous measurement methods, such as nonlinear interferometry and degenerate four-wave mixing. Now it is used in measuring the coefficient of organic optical material and dyes.

In the z-scan experiment, we mostly use beams with a circular profile [1–4], but in fact, laser beams are usually elliptic because of the inherent astigmatism. A novel z-scan technique suitable for elliptic Gaussian beams has been proposed by Tsigaridas et al. [5]. In this paper we focus on theoretical analysis of the z-scan experiment by use of anisotropic Gaussian Schell-model (AGSM) beams. Due to the elliptic profile of the AGSM beam, the circular aperture cannot be used in the far field, so we take the on-axis radiation instead. The expression for the on-axis z-scan transmittance is derived. Based on the expression, we analyze in detail how the degree of spatial coherence and waist width in the \( x \) and \( y \) orientations affect the on-axis z-scan transmittance. The results are helpful for improving the sensitivity of the z-scan experiment.

2. THEORETICAL ANALYSIS

As shown in Fig. 1, an AGSM beam is considered as the incidence beam of the z-scan experimental setup. The cross-spectral density at the front plane of the lens (i.e., \( z = -f \) plane) reads as

\[
W(x_1', y_1', x_2', y_2', z) = I_0 w_{0x} w_{0y} \exp \left( -\frac{x_1'^2 + x_2'^2}{w_{0x}^2} - \frac{y_1'^2 + y_2'^2}{w_{0y}^2} \right) \times \exp \left[ -\frac{(x_1' - x_2')^2}{2 \sigma_{0x}^2} - \frac{(y_1' - y_2')^2}{2 \sigma_{0y}^2} \right],
\]

where \( I_0 \) is a constant factor; \( w_{0x}, w_{0y} \) are the waist widths in the \( x' \) and \( y' \) orientations, respectively; and \( \sigma_{0x}, \sigma_{0y} \) are the correlation lengths in the \( x' \) and \( y' \) orientations, respectively.

Let the AGSM beam travel through the lens and the free space along the \( z \) direction, which can be expressed by

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = \begin{pmatrix}
1 - f + z & f + z \\
1 & -f + 1
\end{pmatrix}.
\]

Ignoring the diffraction of the lens, the cross-spectral density of the beam at the incidence plane of the nonlinear thin sample can be expressed as

\[
W_i(x_1, y_1, x_2, y_2, z) = I_0 w_{0x} w_{0y} \exp \left( -\frac{x_1^2 + x_2^2}{w_{0x}(z)^2} - \frac{y_1^2 + y_2^2}{w_{0y}(z)^2} \right) \times \exp \left[ -\frac{(x_1 - x_2)^2}{2 \sigma_x(z)^2} - \frac{(y_1 - y_2)^2}{2 \sigma_y(z)^2} \right] \times \exp \left[ -\frac{ik(x_1^2 - y_2^2)}{2 R_x(z)} - \frac{ik(y_1^2 - y_2^2)}{2 R_y(z)} \right],
\]
where $w_j(z)$, $\sigma_j(z)$, $R_j(z)$ are the beam width, the correlation length, and the wavefront radius of curvature of the AGSM beam at the $z$ plane, respectively; $k=2\pi/\lambda$ is the wavenumber, and $\lambda$ is the wavelength.

After propagation of the beam through the nonlinear thin sample, the cross-spectral density at the exit surface of the sample, which contains the nonlinear phase distortion, can be expressed as [6]

$$W_\nu(x_1,y_1,x_2,y_2,z)=\exp(-\alpha L)\sum_{m_1=0}^{N}\sum_{m_2=0}^{N} (-1)^{m_1} \frac{(i\Delta \Phi_0)^{m_1}}{m_1!} \frac{w_{0z}w_{0y}}{w_j(z)w_j(z)} \exp \left[ -\frac{(2m_1+1)\lambda^2(x_1^2+y_1^2)+}(m_2+1)\lambda^2y_2^2}{w_j^2(z)} \right] \times \exp \left[ \frac{(x_1-x_2)^2}{2\sigma_j^2(z)} \right] \exp \left[ \frac{-ik(x_1^2-x_2^2)}{2(d-z)} \right] \times \exp \left[ \frac{-ik(y_1^2-y_2^2)}{2R_j(z)} \right].$$

Equation (8) indicates that at the exit surface of the nonlinear sample, the beam can be expressed as a summation of AGSM beams. Each AGSM beam emitted from the exit surface will propagate in free space and reach the output plane, so we can derive the cross-spectral density of the field at the output plane as

$$W_\nu(x_1,y_1,x_2,y_2,z) = \frac{k^2}{2(d-z)^2} \exp(-\alpha L) \left[ \frac{-ik(x_1^2-x_2^2)}{2(d-z)} \right] \times \sum_{m_1=0}^{N}\sum_{m_2=0}^{N} \frac{(i\Delta \Phi_0)^{m_1+m_2}w_{0z}w_{0y}}{m_1!m_2!} \frac{w_j(z)w_j(z)}{\sqrt{(a \cdot b_x-c_x^2)(a \cdot b_y-c_y^2)}} \times \exp \left[ \frac{-k^2(b_x x_1^2+a_x x_1^2-2c_x x_1^2)}{4(d-z)^2(a \cdot b_x-c_x^2)} \right] \exp \left[ \frac{-k^2(b_y y_1^2+a_y y_1^2-2c_y y_1^2)}{4(d-z)^2(a \cdot b_y-c_y^2)} \right].$$

Note: The image contains a schematic diagram of the z-scan setup, which is not transcribed here.
where
\[ a_j = \frac{2m_1 + 1}{w_j^2(z)} + \frac{1}{2\sigma_j^2(z)} + \frac{ik}{2R_j(z) + 2(d - z)}, \]
(10a)
\[ b_j = \frac{2m_2 + 1}{w_j^2(z)} + \frac{1}{2\sigma_j^2(z)} - \frac{ik}{2R_j(z) + 2(d - z)}, \]
(10b)
\[ c_j = \frac{1}{2r_j^2(z)}, \quad j = x, y. \]
(10c)

Letting \( x' = x_1 = x_r, y' = y_1 = y_r \), we can get the irradiance distribution at the output plane:

\[
I_{0}(x', y', z, \Delta \Phi_0) = \left[ \frac{k}{2(d - z)} \right]^2 \exp\left( -aL \right) \sum_{m_1=0}^{N} \sum_{m_2=0}^{N} \frac{(i\Delta \Phi_0)^{m_1+m_2}}{m_1!m_2!} \frac{w_{01}w_{02}}{w_{x}(z)w_{y}(z)} \frac{(-1)^{m_2}}{\sqrt{(a_x b_x - c_x^2)(a_y b_y - c_y^2)}} \exp\left( -\frac{k^2(b_x + a_x - 2c_x)x'^2}{4(d - z)^2(a_x b_x - c_x^2)} \right) \exp\left( -\frac{k^2(b_y + a_y - 2c_y)y'^2}{4(d - z)^2(a_y b_y - c_y^2)} \right). \]
(11)

So the on-axis z-scan transmittance \( T(z) \) can be written as
\[ T(z) = \frac{I_z(x' = y' = 0, z, \Delta \Phi_0)}{I_{0}(x' = y' = 0, z, \Delta \Phi_0 = 0)}. \]
(12)

In the following section, we will analyze the effects of the spatial degree of coherence and waist widths on the z-scan curves by using the above formulas.

### 3. NUMERICAL RESULTS AND ANALYSIS

Based on the formulas of Section 2, we analyze the effects of the spatial degree of coherence and waist widths on the on-axis transmittance. Using the Mathematica software, numerical results of the z-scan are plotted in Figs. 2–8, where the calculation parameters are \( \lambda = 0.532 \times 10^{-3} \text{mm}, w_{01} = 1 \text{mm}, d = 1 \text{m}, f = 200 \text{mm}, \Delta \Phi_0 = 0.01 \) if not otherwise noted.

Figure 2 gives the on-axis z-scan transmittances \( T(z) \) of the AGSM beams with different \( e \) factor \( e = w_{01}/w_{02} \) and the same spatial degree of coherence in both the \( x \) and \( y \) orientations, which are defined by the equations \( a_{0x} = a_{0x}/w_{01}, a_{0y} = a_{0y}/w_{02} \). From Fig. 2(a) we can see that with an increment of the \( e \) factor, the difference between valley and peak becomes larger, the positions of valley and peak become closer, and the curve becomes sharper. The above characters are also shown in Fig. 2(b), in which the peak of the z-scan curve becomes higher and the valley becomes nearly zero with increasing \( e \) factor when the spatial degree of coherence is larger.

Figure 3 gives the on-axis z-scan transmittances \( T(z) \) of the AGSM beams with \( e = 0.5 \) (i.e., \( w_{01} = w_{02} = 1 \text{mm} \)) and the different spatial degree of coherence in the \( x \) and \( y \) orientations. It is shown that the difference between peak and valley in the \( T(z) \) curves increases with an increment of the spatial degree of coherence in either orientation in the case of two orientations having the same beam waist. We can find that the two curves of \( a_{0x} = 0.2, a_{0y} = 0.5 \) and \( a_{0x} = 0.5, a_{0y} = 0.2 \) overlap completely.

The on-axis z-scan transmittances \( T(z) \) of the AGSM beams with \( e = 0.5 \) (i.e., \( w_{01} = 0.5 \text{w}_{02} = 0.5 \text{mm} \)) are plotted in Fig. 4. It can be found from Fig. 4 that the difference between peak and valley in the \( T(z) \) curves increases with an increment of the spatial degree of coherence in either orientation, which is similar to the results of Fig. 3. But there are differences in that the two curves of \( a_{0x} = 1, a_{0y} = 0.5 \) and \( a_{0x} = 0.5, a_{0y} = 1 \) no longer overlap. The positions of valley and peak of \( a_{0x} = 0.5, a_{0y} = 1 \) are closer than...
Fig. 3. On-axis z-scan transmittances $T(z)$ of AGSM beams with $\varepsilon=1$ and different spatial degree of coherence $\alpha_x, \alpha_y$ in the $x$ and $y$ orientations, respectively.

Fig. 4. On-axis z-scan transmittances $T(z)$ of AGSM beams with $\varepsilon=0.5$ and different spatial degree of coherence $\alpha_x, \alpha_y$ in the $x$ and $y$ orientations, respectively.

Fig. 5. On-axis z-scan transmittances $T(z)$ of AGSM beams with $\varepsilon=0.1$, $\alpha_x=\alpha_y=0.5$ for different nonlinear thin samples.

Fig. 6. $\Delta T_{p,v}$ versus $\alpha_x (\alpha_y=\alpha_y)$.

Fig. 7. Transverse intensity distributions of AGSM beams with $\varepsilon=0.1$, $\alpha_x=\alpha_y=1$ at the output plane after the beam has passed through the nonlinear thin medium with different $\Delta \Phi_0$, which is placed at $z=0$. 
Figure 6 shows the difference between the peak and valley of on-axis z-scan transmittance $\Delta T_{p-v} = T_p - T_v$ versus $a_0$, which is equivalent to $a_{0y}$. It is shown that $\Delta T_{p-v}$ increases with an increment of the spatial degree of coherence in the $x$ and $y$ orientations; and while $a_{0y}$ ($=a_{0x}$) becomes larger and larger, $\Delta T_{p-v}$ increases only gradually. Comparing the three curves, we can find that for a given $a_{0x}$ ($=a_{0y}$), the larger the $e$ factor is, the larger the $\Delta T_{p-v}$ is. So we can improve the sensitivity of the $z$-scan experiment by increasing the spatial degree of coherence and the $e$ factor.

In Fig. 7, we plot the transverse intensity distributions of the AGSM beams with $e=0.1$, $a_{0x}=a_{0y}=1$ at the output plane after the beam has passed through the nonlinear thin medium, which is placed at $z=0$. From Fig. 7 we find that for larger $\Delta \Phi_0$, the intensity distributions no longer have Gaussian profile, and the larger $\Delta \Phi_0$ leads to weaker on-axis intensity and larger deviation of the intensity distribution from the Gaussian distribution. Comparing Figs. 7(a) and 7(b), it is found that the intensity distributions in the $x'$ and $y'$ orientations are totally different, because the $e$ factor of AGSM beam is 0.1 (i.e., $w_{0x}=0.1 \text{ mm}$, $w_{0y}=1 \text{ mm}$).

Figure 8 depicts the transverse intensity distributions of the AGSM beams with $e=0.1$ and different $a_{0x}$ ($=a_{0y}$) at the output plane after the beam has passed through the nonlinear thin medium with different $\Delta \Phi_0$, which is placed at $z=0$. It is shown that for a certain $\Delta \Phi_0$, the on-axis intensity becomes weaker with an increment of the spatial degree of coherence in the $x'$ and $y'$ orientations. Moreover, the high coherence leads to larger deviation of the intensity distribution from the Gaussian profile in the $x'$ orientation and to the peak of the intensity moving farther from the axis in the $y'$ orientation.

4. CONCLUSION

The cross-spectral density expression of the AGSM beam passing through the nonlinear thin sample is derived. The effects of the $e$ factor and the spatial degree of coherence in the $x$ and $y$ orientations on the $z$-scan curves are analyzed. We find that the spatial degree of coherence in either orientation has the same effect on the $z$-scan curves for the Gaussian-Schell model beam, while for the AGSM beam, the spatial degree of coherence in the orientation that has a relatively larger beam waist has a much greater effect on the positions of valley and peak and less effect on the difference between valley and peak of the $T(z)$ curves than that in the other orientation.

Figure 5 gives the on-axis $z$-scan transmittances $T(z)$ of the AGSM beams with $e=0.1$, $a_{0x}=a_{0y}=0.5$ for different nonlinear thin samples. It is shown that the curves have two valley–peak features for the AGSM beam with $e=0.1$, $a_{0x}=a_{0y}=0.5$ through the different nonlinear thin samples. That is because the AGSM with much larger difference between waist widths in the $x$ and $y$ orientations has distinctly different focus after propagating through the lens. It is also shown that the difference between valley and peak increases with an increment of $\Delta \Phi_0$, which is related to the nonlinear refractive index $\gamma$ through $\Delta \Phi_0 = \gamma k L_{eff}$. 

Fig. 8. Transverse intensity distributions of AGSM beams with $e=0.1$ and different $a_{0x}$ ($=a_{0y}$) at the output plane after the beam has passed through the nonlinear thin medium, which is placed at $z=0$. 

those of $a_{0x}=1$, $a_{0y}=0.5$, while the difference between peak and valley of $a_{0x}=1$, $a_{0y}=0.5$ is larger than that of $a_{0x}=0.5$, $a_{0y}=1$. So we can conclude that the spatial degree of coherence in the orientation that has a relatively larger beam waist has a much greater effect on the positions of valley and peak and less effect on the difference between valley and peak of the $T(z)$ curves than that in the other orientation.
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REFERENCES